POST-GRADUATE COURSE

Term End Examination — June, 2022/December, 2022

MATHEMATICS

Paper-9B(ii) : MATHEMATICAL MODELS IN ECOLOGY

(Applied Mathematics)

(Spl. Paper)

Time : 2 hours] [Full Marks : 50

Weightage of Marks : 80%

Special credit will be given for accuracy and relevance in the answer. Marks will be deducted for incorrect spelling, untidy work and illegible handwriting. The marks for each question has been indicated in the margin.

Answer Question No. **1** and any *four* from the rest :

1. Answer any *five* questions : $2 \times 5 = 10$

   a) Write down the logistic model of population growth explaining the different terms involved in it.

   b) Describe Routh-Hurwitz criterion.

   c) What do you mean by stability of equilibrium of a system ?

   d) What is mutualism ?

   e) Write down the types of delay population models with examples.

   f) Solve the Gompertz growth model $\frac{dx}{dt} = r_0 e^{-\alpha t} x$ with initial condition $x(0) = x_0$.

   g) Describe ecosystem model with one functional group having constant primary production.

2. Write down the classical predator-prey Lotka-Volterra model explaining the different terms involved in it. Non-dimensionalize the system. Hence analyze the equilibrium points and their stability. 10

3. Consider the following two-dimensional system :

$$\frac{dx}{dt} = y - x$$

$$\frac{dy}{dt} = \frac{5x^2}{4+x^2} - y$$

Determine the equilibrium points. Hence determine the local stability of each positive equilibrium point and classify the equilibrium points. 10

4. Describe Lotka-Volterra classical competition model. What is zero-growth isoclines ? Discuss the cases depending on the relative position of intercepts of two zero-growth isoclines. 10

5. Consider the following linear second-order homogeneous difference equation : $x_{n+1} = x_n + x_{n-1}$. Find out its general solution. Identity the dominant eigenvalue with reason. How is the model related to the Golden mean (1·618033 ...) ? 10

6. a) Find the non-negative equilibrium of a population governed by

$$x_{n+1} = \frac{2x_n^2}{x_n^2 + 2}$$

and check the stability. 5

b) Consider the following equation :

$$x_{n+1} = rx_n(1 - x_n).$$

Investigate the stability of the steady-state. 5

7. a) Write down the logistic model for a single-species population. Hence explain the concepts of carrying capacity and intra-species competition. 4

b) If a population in governed by the logistic growth equation and is subject to constant harvesting, write down the model equation for this population. Find the equilibrium points of the model and discuss their stability. 6